Probabilistic Graphical Models

Relationship between the directed & undirected models

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Learning Objective

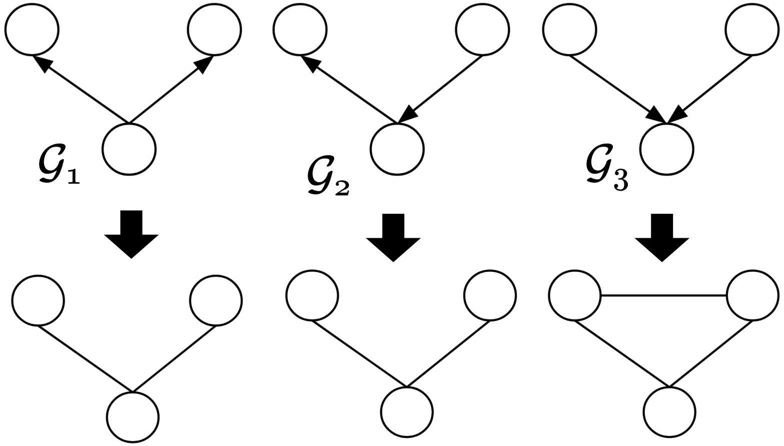
understand the relationship between CIs
in directed and undirected models.

convert

- Markov network $\Rightarrow$ Bayes-net
- Markov network $\Leftarrow$ Bayes-net

1. From Bayesian to Markov networks

build an I-map for the following



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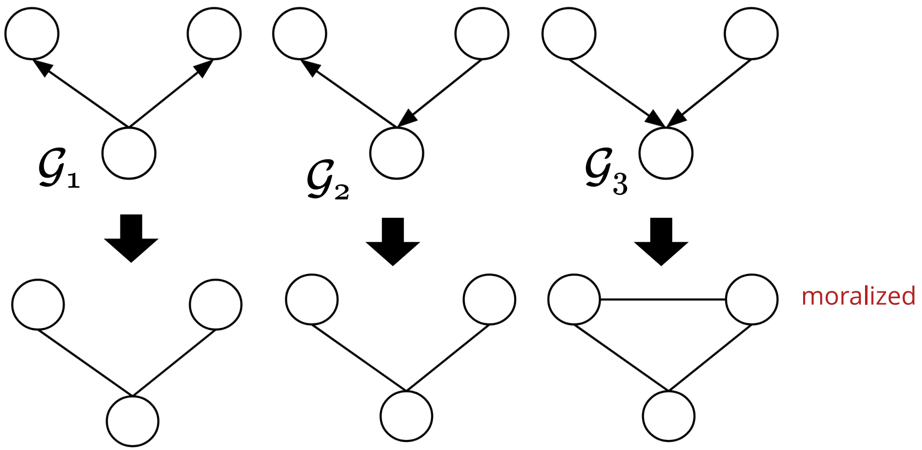

$$\mathcal{I}(\mathcal{M}[\mathcal{G}_1]) = \mathcal{I}(\mathcal{G}_1) \qquad \mathcal{I}(\mathcal{M}[\mathcal{G}_3]) \subseteq \mathcal{I}(\mathcal{G}_3)$$

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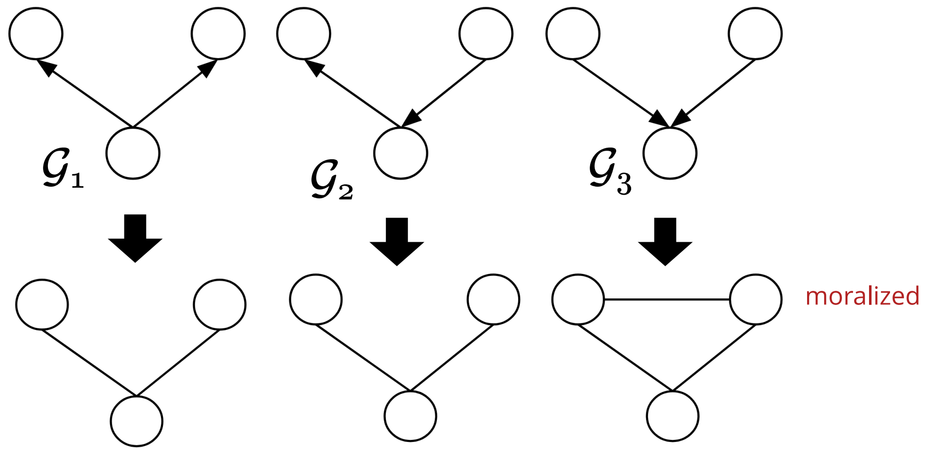

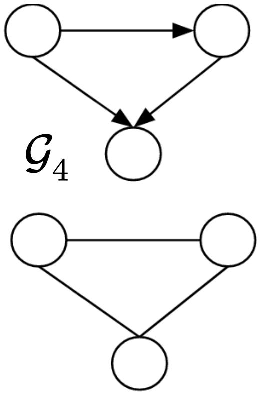

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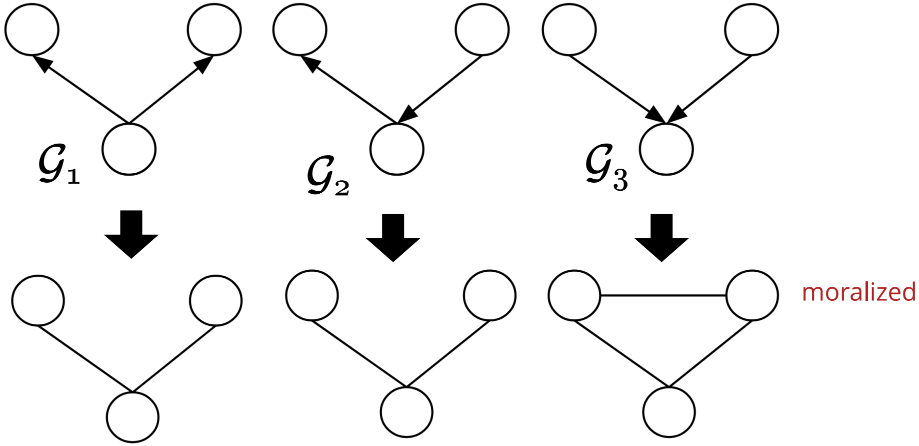

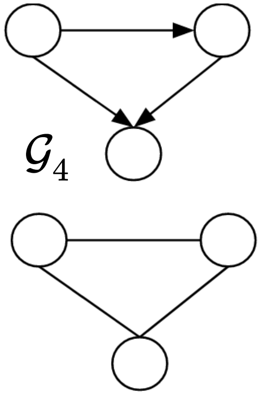

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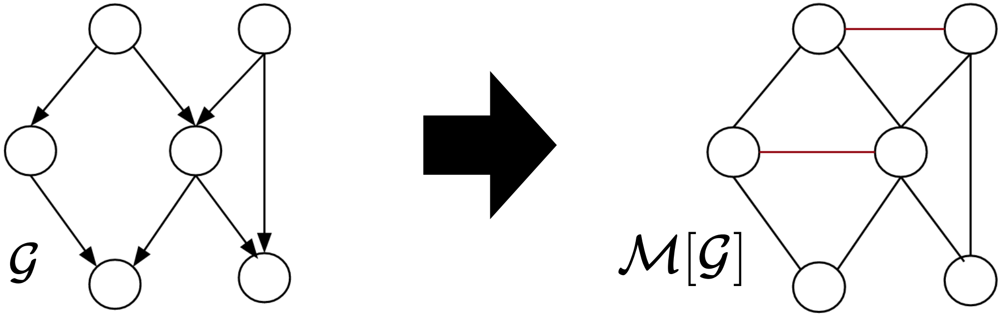
Moralize $\mathcal{G} \to \mathcal{M}(\mathcal{G})$:connect parents keep the skeleton

From Bayesian to Markov networks

moralize & keep the skeleton

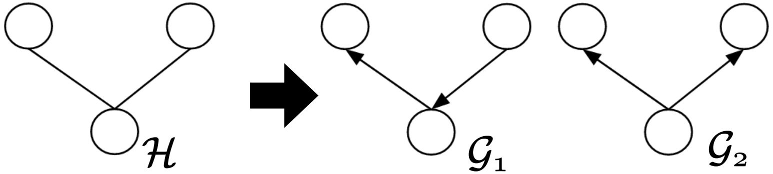


for moral $\mathcal{G}$, we get a perfect map $\mathcal{I}(\mathcal{M}[\mathcal{G}]) = \mathcal{I}(\mathcal{G})$

- *directed and undirected CI tests are equivalent*

2. From Markov to Bayesian networks

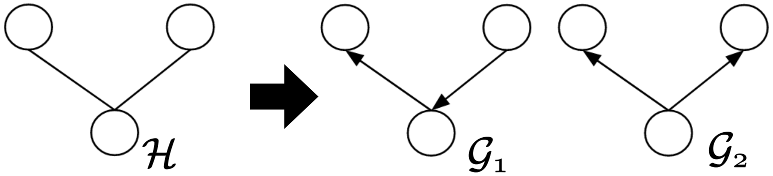
`minimal examples 1.`



$$\mathcal{I}(\mathcal{G}_1) = \mathcal{I}(\mathcal{G}_2) = \mathcal{I}(\mathcal{H})$$

2. From Markov to Bayesian networks

minimal examples 1.



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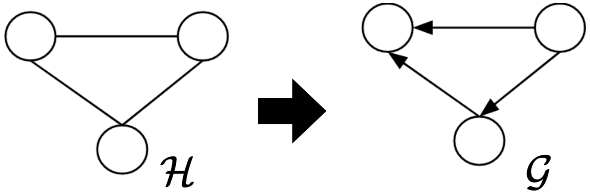
minimal examples 2.



$$\mathcal{I}(\mathcal{G}) = \mathcal{I}(\mathcal{H})$$

From Markov to Bayesian networks

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minimal examples 3.
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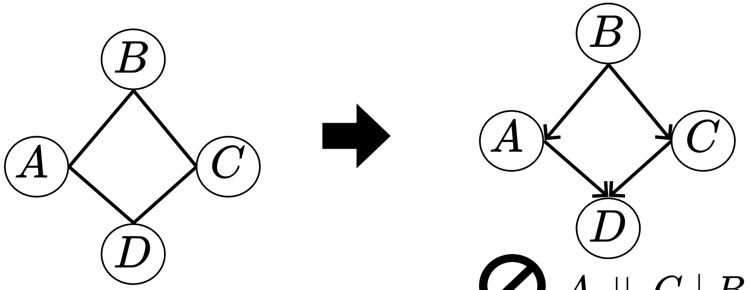
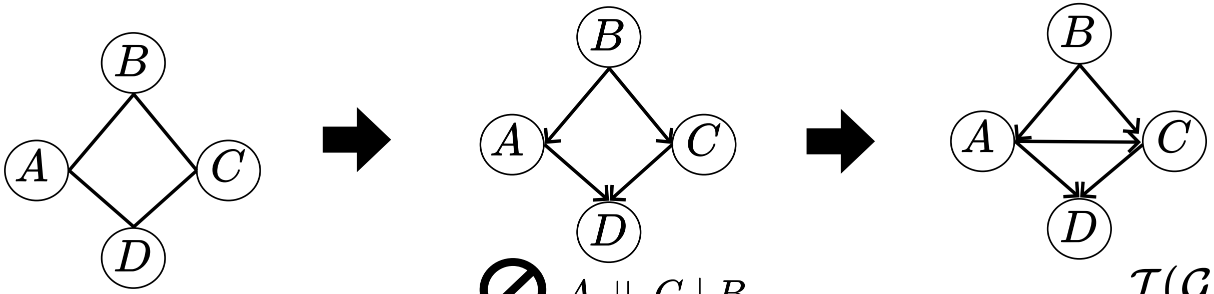


$A \perp\!\!\!\perp C \mid B$

From Markov to Bayesian networks

`minimal examples 3.`



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From Markov to Bayesian networks

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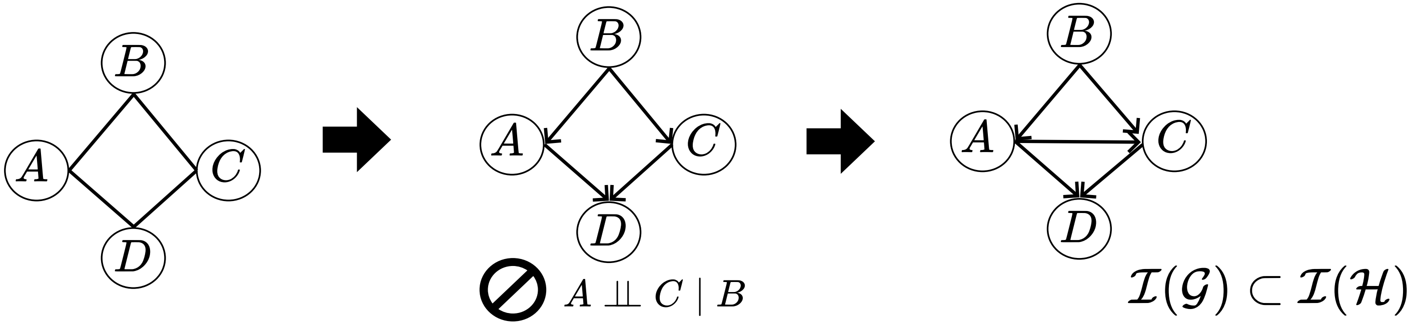

- if adding direction creates immoralities, we have to moralize

From Markov to Bayesian networks

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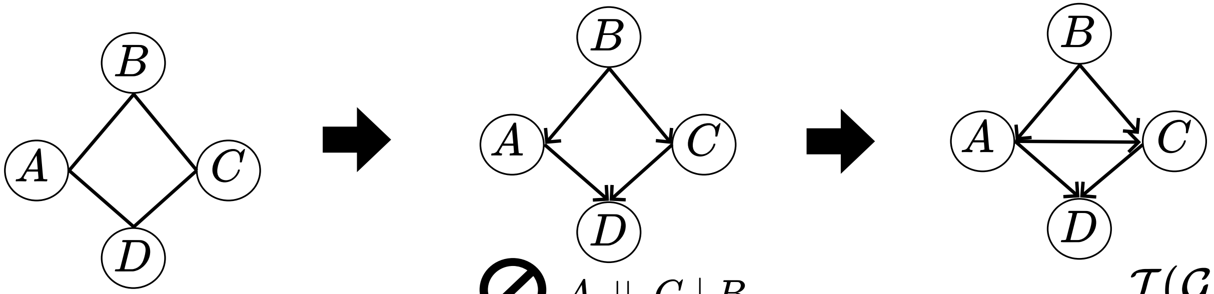
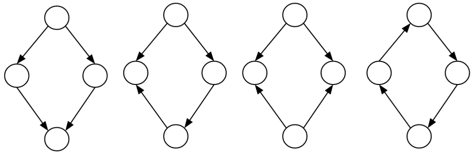


$A \perp\!\!\!\perp C \mid B$

$\mathcal{I}(\mathcal{G}) \subset \mathcal{I}(\mathcal{H})$

- if adding direction creates immoralities, we have to moralize
- any non-triangulated loop > 3 creates immorality
- have to triangulate the loops



From Markov to Bayesian networks

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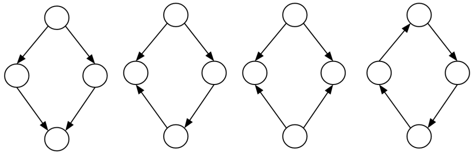
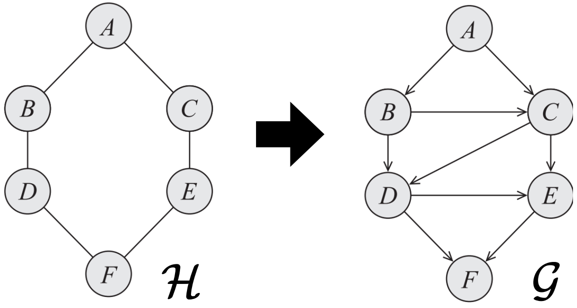
- have to **triangulate** the loops



examples 4.



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$\mathcal{H}$

From Markov to Bayesian networks

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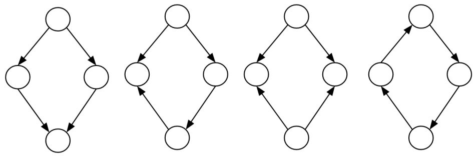

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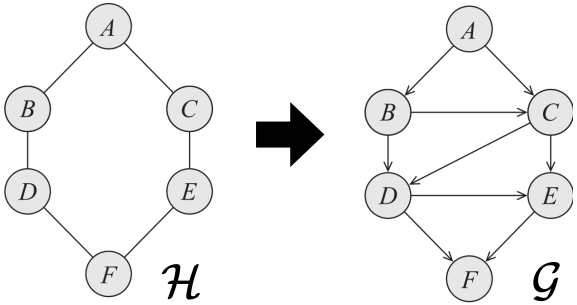


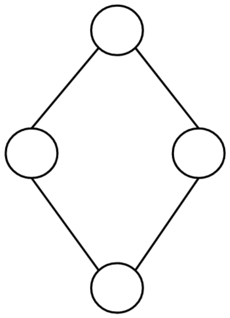
$\mathcal{I}(\mathcal{G}) \subset \mathcal{I}(\mathcal{H})$

a graph $\mathcal{H}$ is **chordal** if all loops >3 have a "chord"

Chordal = Markov $\cap$ Bayesian networks

$\mathcal{H}$ is not chordal, then $\mathcal{I}(\mathcal{G}) \neq \mathcal{I}(\mathcal{H})$ for **every** $\mathcal{G}$

- no *perfect MAP* in the form of Bayes-net



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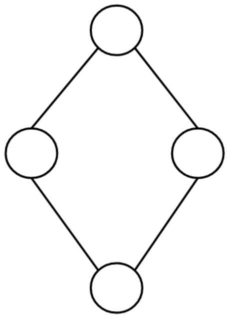
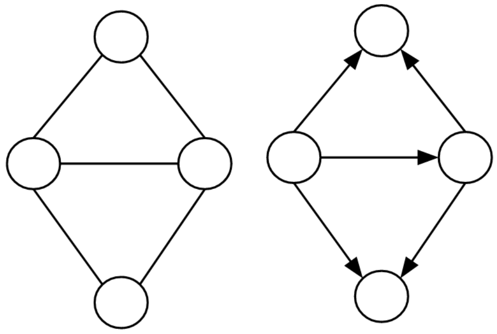


$\mathcal{H}$ is chordal, then $\mathcal{I}(\mathcal{G}) = \mathcal{I}(\mathcal{H})$ for **some** $\mathcal{G}$

- has a Bayes-net perfect map



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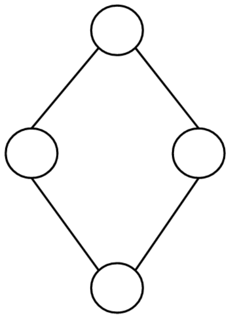

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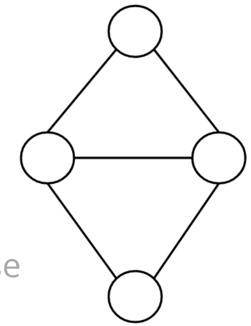
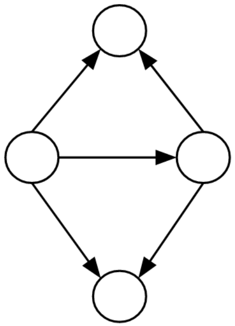
need clique-trees to build these





directed

- parameter-estimation is easy
- can represent causal relations
- better for encoding expert domain knowledge

undirected

- simpler CI semantics
- less interpretable form for local factors
- less restrictive in structural form (loops)

Summary

- directed to undirected:
  - moralize
- undirected to directed:
  - triangulate
- Chordal graphs = Markov $\cap$ Bayesian networks
  - p-maps in both directions

Bonus

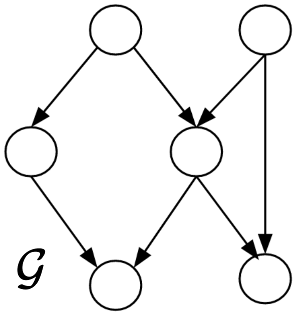
From Bayesian to Markov networks

alternative approach

- in both directed and undirected models

$$X_i \perp \text{every other var.} \mid MB(X_i)$$

- connect each node to its **Markov blanket**



children +
parents +
parents of children

From Bayesian to Markov networks

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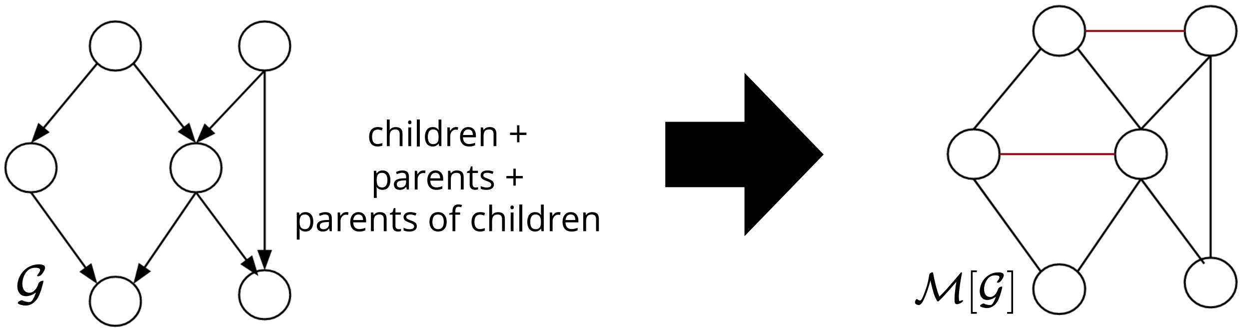

- gives the same moralized graph