Graphical Models

Relationship between the directed & undirected models

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Winter 2018

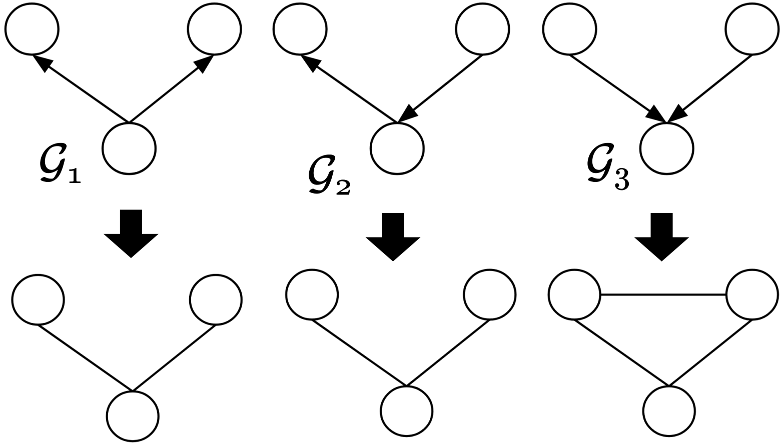
Two directions

Markov network $\Rightarrow$ Bayes-net

Markov network $\Leftarrow$ Bayes-net

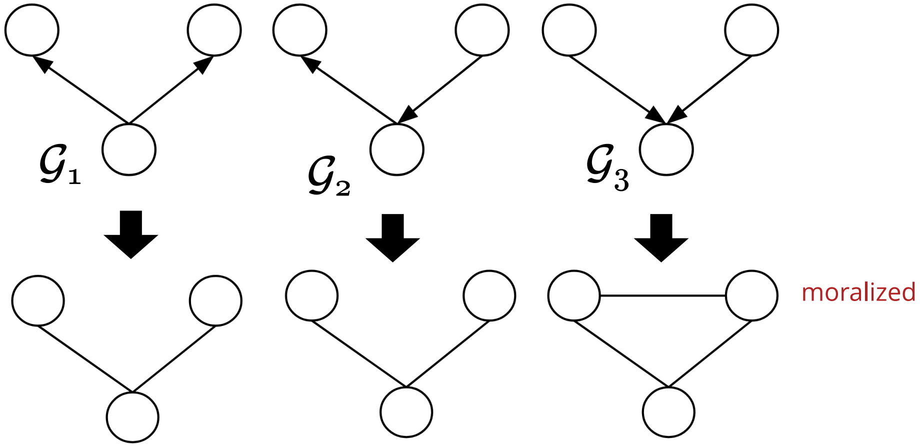
From Bayesian to Markov networks

build an I-map for the following



From **Bayesian** to **Markov** networks

build an I-map for the following



$$\mathcal{I}(\mathcal{M}[\mathcal{G}_1]) = \mathcal{I}(\mathcal{G}_1) \qquad \mathcal{I}(\mathcal{M}[\mathcal{G}_3]) \subseteq \mathcal{I}(\mathcal{G}_3)$$

From Bayesian to Markov networks

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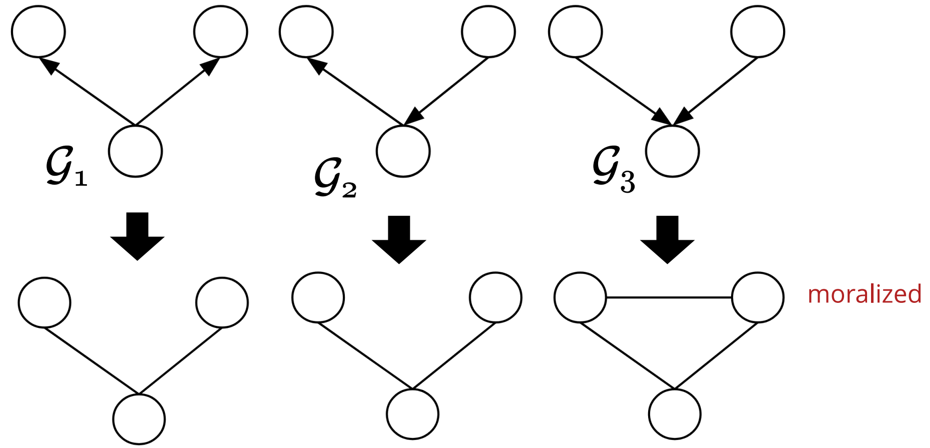

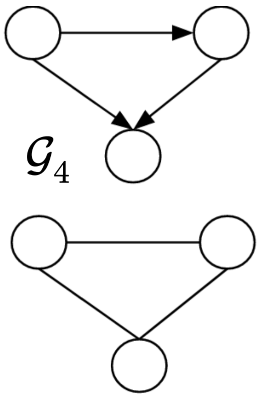

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From Bayesian to Markov networks

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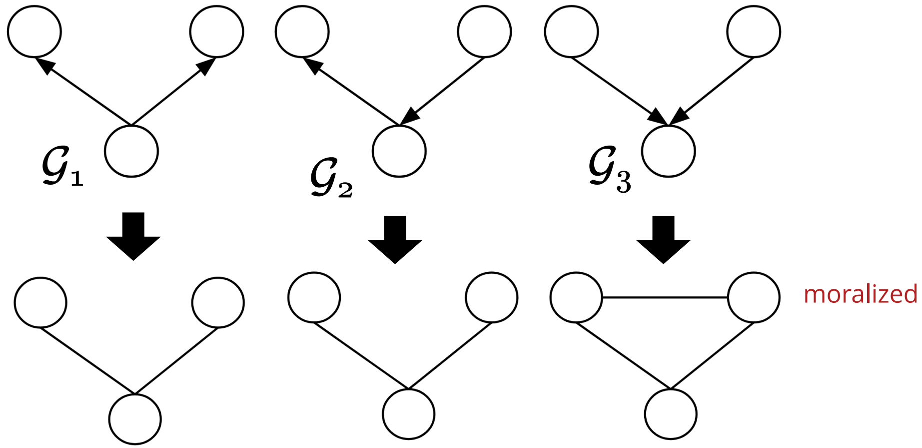

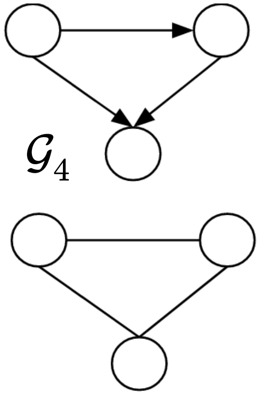

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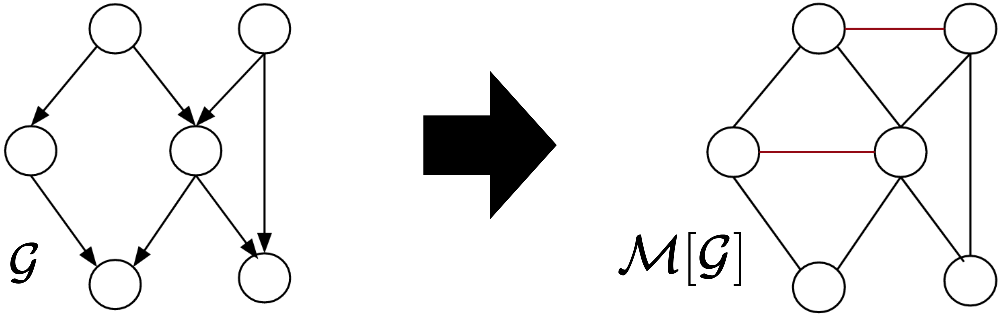
moralize & keep the skeleton

From Bayesian to Markov networks

moralize & keep the skeleton



for moral $\mathcal{G}$, we get a perfect map $\mathcal{I}(\mathcal{M}[\mathcal{G}]) = \mathcal{I}(\mathcal{G})$

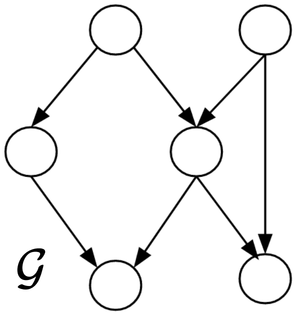
- *directed and undirected CI tests are equivalent*

From Bayesian to Markov networks

- in both directed and undirected models

$$X_i \perp \text{every other var.} \mid MB(X_i)$$

- connect each node to its **Markov blanket**



children +
parents +
parents of children

From Bayesian to Markov networks

- in both directed and undirected models

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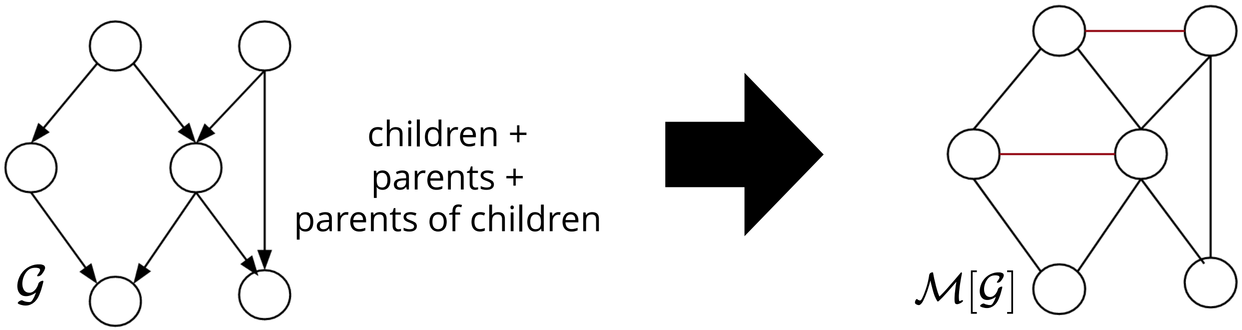
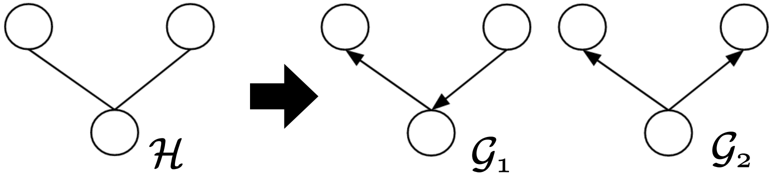


- gives the same moralized graph

From Markov to Bayesian networks

minimal examples 1.



$$\mathcal{I}(\mathcal{G}_1) = \mathcal{I}(\mathcal{G}_2) = \mathcal{I}(\mathcal{H})$$

From Markov to Bayesian networks

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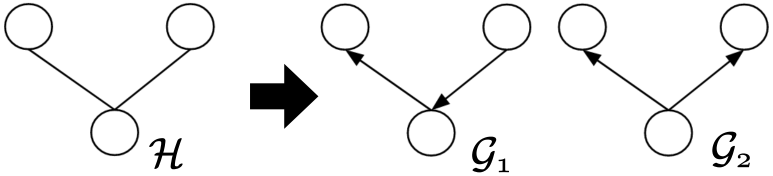

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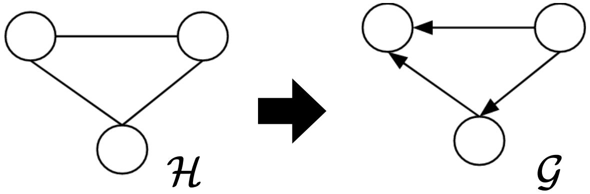

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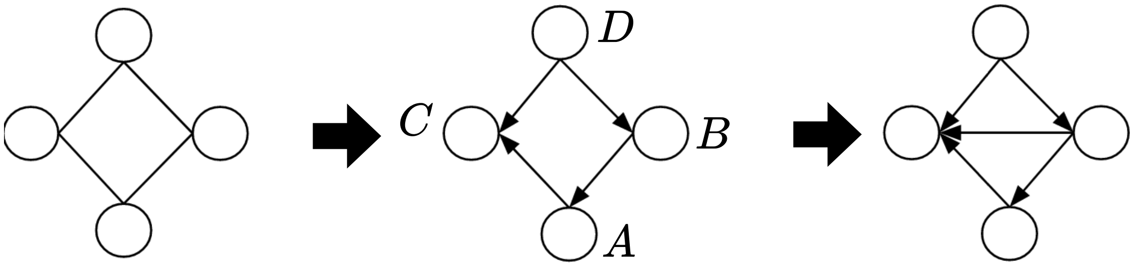
From Markov to Bayesian networks

minimal examples 3.



From Markov to Bayesian networks

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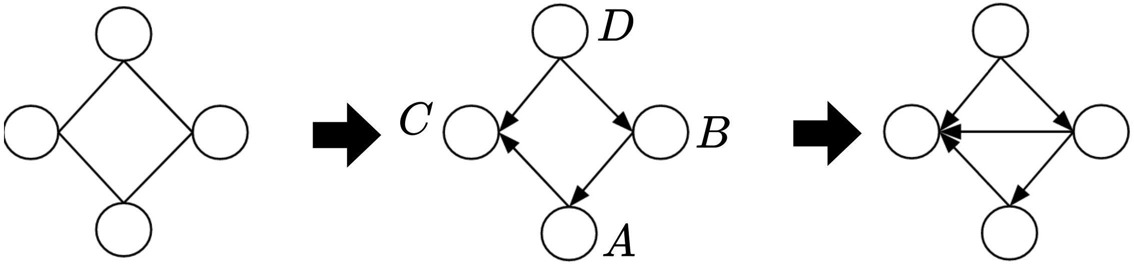
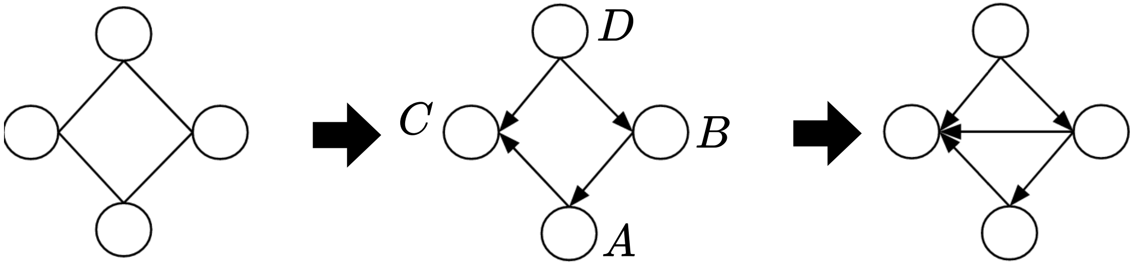


$\oslash$ $B \perp C \mid A$

$\mathcal{I}(\mathcal{G}) \subset \mathcal{I}(\mathcal{H})$

From Markov to Bayesian networks

minimal examples 3.



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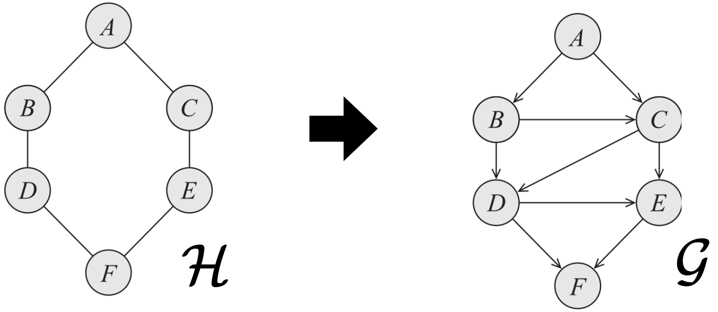
examples 4.



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From Markov to Bayesian networks

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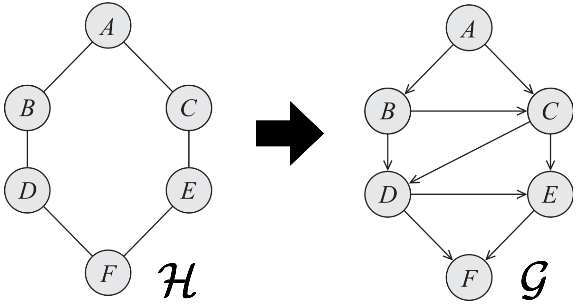


$\mathcal{I}(\mathcal{G}) \subset \mathcal{I}(\mathcal{H})$

build a **minimal** I-map from CIs in $\mathcal{H}$:

- pick an ordering - e.g., A,B,C,...,F
- select a minimal parent set

- have to triangulate the loops
- therefore, $\mathcal{G}$ is **chordal**

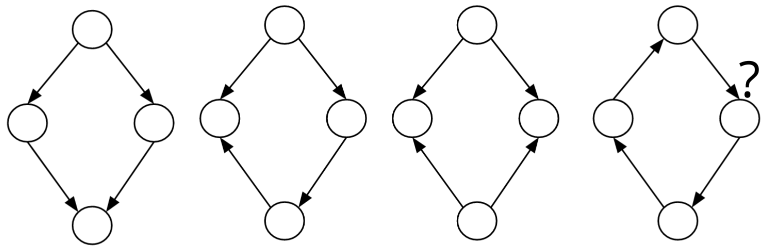
loops of size >3 have *chords*

From **Markov** to **Bayesian** networks

alternatively

$\mathcal{I}(\mathcal{G}) \subseteq \mathcal{I}(\mathcal{H}) \Rightarrow \mathcal{G}$ cannot have any immoralities

any **non-triangulated** loop of size 4 (or more) will have immoralities



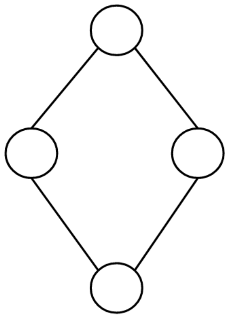
therefore, $\mathcal{G}$ is **chordal**

loops of size >3 have *chords*

Chordal = Markov $\cap$ Bayesian networks

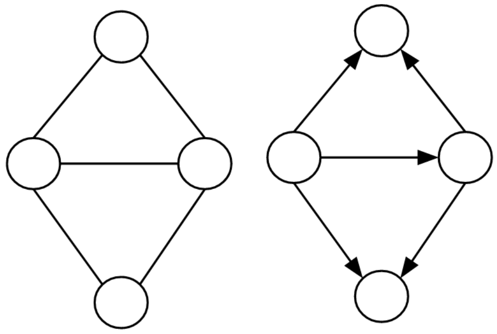
$\mathcal{H}$ is not chordal, then $\mathcal{I}(\mathcal{G}) \neq \mathcal{I}(\mathcal{H})$ for **every** $\mathcal{G}$

- no *perfect MAP* in the form of Bayes-net



$\mathcal{H}$ is chordal, then $\mathcal{I}(\mathcal{G}) = \mathcal{I}(\mathcal{H})$ for **some** $\mathcal{G}$

- has a Bayes-net perfect map



directed

- parameter-estimation is easy
- can represent causal relations
- better for encoding expert domain knowledge

undirected

- simpler CI semantics
- less interpretable form for local factors
- less restrictive in structural form (loops)

Summary

- Directed to undirected:
  - moralize
- Undirected to directed:
  - the result will be chordal
- Chordal graphs = Markov $\cap$ Bayesian networks
  - P-maps in both directions